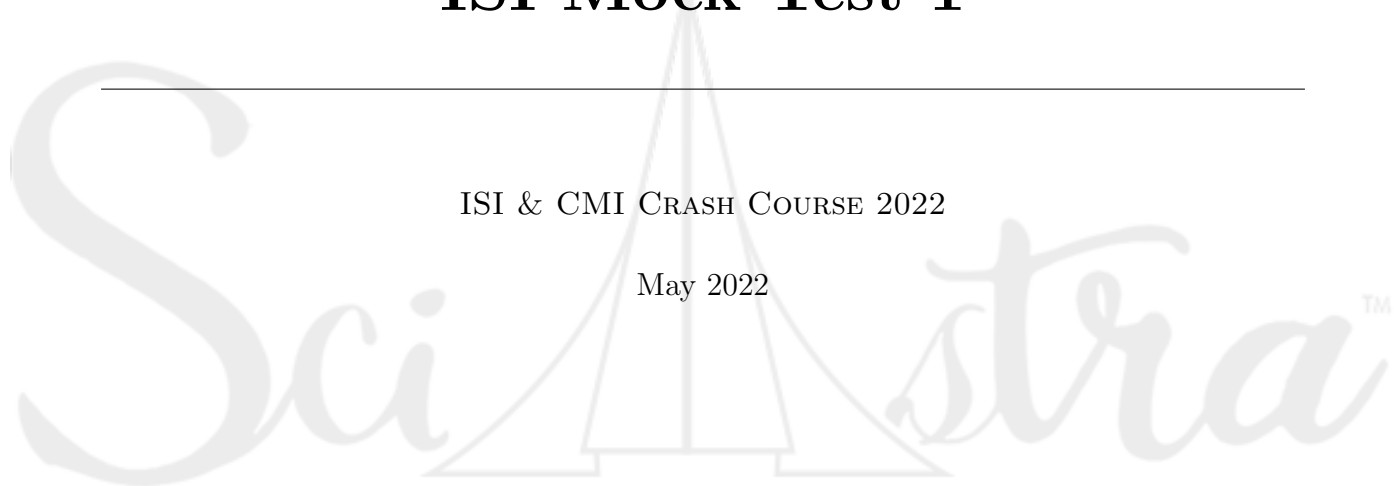


ISI Mock Test 1

ISI & CMI CRASH COURSE 2022

May 2022



Instructions

The examination is comprised of 30 objective (**UGA**) and 8 subjective (**UGB**) questions. Attempt however many you can in the given time.

0.1 Marking Scheme for UGA

- For every correct answer, you shall receive **+4** points.
- For every incorrect answer, you shall receive **0** (zero) points.
- For every unattempted question, you shall receive **+1** points.

0.2 Marking Scheme for UGB

Each question carries 10 marks. Write your answers clearly and state all results used. In case of partially correct answers, partial credit may be awarded, based upon the examiner's discretion.

0.3 Timings

- **UGA:** 10:00 - 12:00 IST, 2 hours.
- **UGB:** 14:00 - 16:00 IST, 2 hours.

UGA

1. Let ABC be a triangle such that $AB = BC$. Let F be the midpoint of AB and X be a point on BC such that $FX \perp AB$. If $BX = 3XC$, then the ratio $\frac{BC}{AC}$ equals

- (a) $\sqrt{3}$
- (b) $\sqrt{2}$
- (c) $\sqrt{\frac{3}{2}}$
- (d) 1

2. In the real number system the equation $\sqrt{x+3-4\sqrt{x-1}} + \sqrt{x+8-6\sqrt{x-1}} = 1$ has

- (a) No solution
- (b) Exactly two distinct solutions
- (c) Exactly four distinct solutions
- (d) Infinitely many solutions

3. Let $n \in \mathbb{N}, a \in \mathbb{R}$. The number of zeroes of $x^{2n+1} - (2n+1)x + a = 0$ in the interval $[-1, 1]$ is

- (a) 2 if $a > 0$
- (b) 2 if $a < 0$
- (c) At most one $\forall a$
- (d) At least three $\forall a$

4. The number of solutions to the equation

$$\cos^4 x + \frac{1}{\cos^2 x} = \sin^4 x + \frac{1}{\sin^2 x}$$

in the interval $[0, 2\pi]$ is

- (a) 6
- (b) 4
- (c) 2
- (d) 0

5. Which of the following is a possible domain of the function $f(x) = \log_{\{x\}} \lfloor x \rfloor + \log_{\lfloor x \rfloor} \{x\}$?

- (a) $(0, 1)$
- (b) $(1, 2)$
- (c) $(2, 3)$

- (d) $(3, 5)$
6. The minimum distance between a point on $y = e^x$ and a point on $y = \log_e x$ is
- $\frac{1}{\sqrt{2}}$
 - $\sqrt{2}$
 - $\sqrt{3}$
 - $2\sqrt{2}$
7. The maximum possible value of $x^2 + y^2 - 4x - 6y$, x, y are real, subject to the condition $|x + y| + |x - y| = 4$
- 12
 - 28
 - 72
 - does not exist
8. The maximum value of $3^x + 5^x - 9^x + 15^x - 25^x$ as x varies over reals, belongs to
- $(3, 5)$
 - $(0, 2)$
 - $(9, 25)$
 - $(5, 9)$
9. Let a, b, c, d be numbers in the set $\{1, 2, 3, 4, 5, 6\}$ such that the curves $y = 2x^3 + ax + b$ and $y = 2x^3 + cx + d$ have no point in common. The maximum possible value of $(a - c)^2 + b - d$ is
- 0
 - 5
 - 30
 - 36
10. $f : \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = 2x + |\cos x|$, is
- one-one and into
 - one-one and onto
 - many-one and into
 - many-one and onto
11. Let $f\left(x + \frac{1}{x}\right) = x^2 + \frac{1}{x^2}$, $x \neq 0$ then $f(x)$ equals
- $x^2 - 2 \quad \forall \quad x$
 - $x^2 - 2 \quad \forall \quad |x| > 2$
 - $x^2 - 2 \quad \forall \quad |x| < 2$
 - none of these
12. If $|z - 2 + i| = |z| \left| \sin\left(\frac{\pi}{4} - \arg(z)\right) \right|$, then the locus of z is
- pair of straight lines

- (b) circle
(c) parabola
(d) ellipse
13. Let $f(n) = \sum_{r=1}^n r^2 \binom{n}{r}^2$. Then $f(5) =$
- (a) $\binom{8}{4}$
(b) $25 \binom{8}{4}$
(c) $25 \binom{8}{5}$
(d) None of these
14. The area of the region bounded by $|x| + |y| + |x + y| \leq 2$ is
- (a) 2
(b) 3
(c) 4
(d) 6
15. Limit $\lim_{x \rightarrow \infty} \int_0^x e^{t^3 - x^3} dt$ equals
- (a) $\frac{1}{3}$
(b) 2
(c) ∞
(d) $\frac{2}{3}$
16. The range of polynomial $p(x) = 4x^3 - 3x$ as x varies over interval $(-\frac{1}{2}, \frac{1}{2})$ is
- (a) $[-1, 1]$
(b) $(-1, 1]$
(c) $(-1, 1)$
(d) $(-\frac{1}{2}, \frac{1}{2})$
17. The remainder when $3^{79^{79}}$ is divided by 79
- (a) 0
(b) 1
(c) 2
(d) 78
18. If $0 < x < \frac{\pi}{2}$, then
- (a) $\cos(\cos(x)) > \sin(x)$
(b) $\sin(\sin(x)) > \sin(x)$
(c) $\sin(\cos(x)) > \cos(x)$

(d) $\cos(\sin(x)) > \sin(x)$

19. Consider an array of m rows and n columns obtained by arranging the first mn positive integers in some order. Let b_i be the maximum of the numbers in the i th row and c_j be the minimum of the numbers in the j th column. If

$$b = \min_{1 \leq i \leq m} b_i$$

and

$$c = \max_{1 \leq j \leq n} c_j$$

then which of the following statements are necessarily true?

- (a) $m \leq c$
 - (b) $n \geq b$
 - (c) $c \geq b$
 - (d) $c \leq b$
20. Let $S_n = \sum_{k=1}^n k$. The numbers $S_1, S_2, \dots, S_{99}$ are written on 99 cards. The probability of drawing a card with an even number written on it is
- (a) $\frac{1}{2}$
 - (b) $\frac{49}{100}$
 - (c) $\frac{49}{99}$
 - (d) $\frac{49}{98}$
21. A man tosses a coin 10 times, scoring 1 point for each head and 2 points for each tail. Let $P(\kappa)$ be the probability of scoring at least κ points. The largest value of κ such that $P(\kappa) > \frac{1}{2}$ is
- (a) 14
 - (b) 15
 - (c) 16
 - (d) 17
22. Let $f(x) : [2, \infty] \rightarrow \mathbb{N}$ be defined by $f(x) =$ the largest prime factor of $\lfloor x \rfloor$. Then $\int_2^8 f(x) dx$ is equal to
- (a) 17
 - (b) 22
 - (c) 23
 - (d) 25
23. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a continuous function satisfying $f(x) = x + \int_0^x f(t) dt$, for all $x \in \mathbb{R}$. Then the number of elements in the set $S = \{x \in \mathbb{R}; f(x) = 0\}$ is
- (a) 1
 - (b) 2
 - (c) 3

- (d) 4
24. Let $p(x)$ be a polynomial such that $p(x) - p'(x) = x^n$, where n is a positive integer. Then $p(0)$ equals
- $n !$
 - $(n - 1)!$
 - $\frac{1}{n!}$
 - $\frac{1}{(n-1)!}$
25. The equation $x^3y + xy^3 + xy = 0$ represents
- a circle
 - a circle and a pair of straight lines
 - a rectangular hyperbola
 - a pair of straight lines
26. For any integer $n \geq 1$, define $a_n = \frac{1000^n}{n!}$. Then the sequence $\{a_n\}$
- does not have a maximum
 - attains maximum at exactly one value of n
 - attains maximum at exactly two values of n
 - attains maximum for infinitely many values of n .
27. Let ABCD be a unit square. Four points E, F, G and H are chosen on the sides AB, BC, CD and DA respectively. The lengths of the sides of the quadrilateral EF GH are α, β, γ and δ . Which of the following is always true?
- $1 \leq \alpha^2 + \beta^2 + \gamma^2 + \delta^2 \leq 2\sqrt{2}$
 - $2\sqrt{2} \leq \alpha^2 + \beta^2 + \gamma^2 + \delta^2 \leq 4\sqrt{2}$
 - $2 \leq \alpha^2 + \beta^2 + \gamma^2 + \delta^2 \leq 4$
 - $\sqrt{2} \leq \alpha^2 + \beta^2 + \gamma^2 + \delta^2 \leq 2 + \sqrt{2}$
28. The number of functions $f : [0, 1] \rightarrow [0, 1]$ such that $f(x) < x^2$ for all x and $\int_0^1 f(x)dx = \frac{1}{3}$ are
- 0
 - 1
 - 2
 - ∞
29. Let $f : R \rightarrow R$ be a continuous function such that $f(x+1) = \frac{1}{2}f(x)$ for all $x \in R$, and let $a_n = \int_0^n f(x)dx$ for all integers $n \geq 1$. Then:
- $\lim_{n \rightarrow \infty} a_n$ exists and equals $\int_0^1 f(x)dx$.
 - $\lim_{n \rightarrow \infty} a_n$ does not exist.
 - $\lim_{n \rightarrow \infty} a_n$ exists if and only if $\left| \int_0^1 f(x)dx \right| < 1$.
 - $\lim_{n \rightarrow \infty} a_n$ exists and equals $2 \int_0^1 f(x)dx$

30. Suppose $f(x)$ is a twice differentiable function on $[a, b]$ such that $f(a) = 0 = f(b)$ and $x^2 f''(x) + 4x f'(x) + 2f(x) > 0 \quad \forall x \in (a, b)$

Then,

- (a) f is negative for all $x \in (a, b)$.
- (b) f is positive for all $x \in (a, b)$.
- (c) $f(x) = 0$ for exactly one $x \in (a, b)$.
- (d) $f(x) = 0$ for at least two $x \in (a, b)$

